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Analyzing Symmetry in Photonic Band Structure of Gyro-magnetic Photonic Crystals

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ABSTRACT

In the band structure analysis of photonic crystals it is normally assumed that the full photonic gaps could be found by scanning high-symmetry paths along the edges of Irreducible Brillouin Zones (IBZ). We have recently shown [1] that this assumption is wrong in general for sufficiently symmetry breaking geometries, so that the IBZ is exactly half of the complete BZ. That minimal required symmetry arises from the requirement on time-reversal symmetry. In this paper we show that even that requirement might be broken by using gyro-magnetic materials in the composition of photonic structures we can observe that the IBZ extends fully to the boundaries of the complete BZ, that is IBZ must be as the same as BZ.

Keywords: symmetry, photonic crystal, plane wave expansion, band structure

1. INTRODUCTION

It is customary to traverse boundary of irreducible BZ for describing photonic band structure. Moosavi Mehr and Khorasani [1] showed that this results in correct description only when there is C_{4v} symmetry. Time reversal symmetry introduces a characteristic that we can just analyze at most half of BZ for completely describing photonic band structure. In this article we present structures that does not hold any symmetry even time reversal symmetry. Thus in these structures we must analyze whole BZ. At first we derive eigen-value equation for obtaining eigen-frequencies. Then we present structures that do not hold any symmetry.

Recently, it has been shown [2-4] that time-reversal symmetry can be broken in certain photonic structures by using magneto-optical materials. These class of materials under proper fabrication can resemble topological surface states in condensed matter systems, which similarly violate time-reversal symmetry. However, usage of magneto-optical materials is still insufficient to completely remove some of the basic symmetry properties of periodic structures. A non-trivial symmetry group with an order above unity will lead to an irreducible Brillouin Zone which is an integer divider of the First Brillouin Zone.

In this paper, for the first time we show that it is possible to totally remove the remaining symmetries in the first Brillouin Zone to obtain an indentical Irreducible Zone which spans the whole Brillouin Zone. It is readily therefore concluded that for certain structures the photonic band structure should be examined over the complete Brillouin Zone and not only on the high-symmetry paths to obtain the true photonic gaps.

2. MATHEMATICAL EQUATIONS AND EIGEN-VALUE EQUATION

For a TM-polarized wave in gyro-magnetic photonic crystal the following relations hold [3,4]:

$$\mu(\boldsymbol{r}) = \begin{bmatrix} \mu(\boldsymbol{r}) & i\rho(\boldsymbol{r}) & 0 \\ -i\rho(\boldsymbol{r}) & \mu(\boldsymbol{r}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

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$$\mu^{-1}(\boldsymbol{r}) = \begin{bmatrix} \tilde{\mu}^{-1}(\boldsymbol{r}) & i\eta(\boldsymbol{r}) & 0 \\ -i\eta(\boldsymbol{r}) & \tilde{\mu}^{-1}(\boldsymbol{r}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\eta(\boldsymbol{r}) = \frac{-\rho(\boldsymbol{r})}{\mu^2(\boldsymbol{r}) - \rho^2(\boldsymbol{r})} \tag{3}$$

$$\tilde{\mu}^{-1}(\boldsymbol{r}) = \frac{\mu(\boldsymbol{r})}{\mu^2(\boldsymbol{r}) - \rho^2(\boldsymbol{r})} \tag{4}$$

$$\frac{1}{\varepsilon(\boldsymbol{r})} \nabla \times \mu^{-1}(\boldsymbol{r}) \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \boldsymbol{E} \tag{5}$$

$$\frac{1}{\varepsilon(\boldsymbol{r})} \nabla \times \frac{1}{\tilde{\mu}(\boldsymbol{r})} \nabla \times \boldsymbol{E} + \frac{1}{\varepsilon(\boldsymbol{r})} \nabla \times \begin{bmatrix} 0 & j\eta(\boldsymbol{r}) \\ -j\eta(\boldsymbol{r}) & 0 \end{bmatrix} \nabla \times \boldsymbol{E} = \frac{\omega^2}{c^2} \boldsymbol{E} \tag{6}$$

$$\boldsymbol{E} = E_z \hat{z} \tag{7}$$

$$A = \nabla \times \frac{1}{\tilde{\mu}(\boldsymbol{r})} \nabla \times \boldsymbol{E} \tag{8}$$

$$B = \nabla \times \begin{bmatrix} 0 & j\eta(\boldsymbol{r}) \\ -j\eta(\boldsymbol{r}) & 0 \end{bmatrix} \nabla \times \boldsymbol{E} \tag{9}$$

$$\frac{\partial E_z}{\partial z} = 0 \tag{10}$$

$$\begin{bmatrix} 0 & j\eta(\boldsymbol{r}) \\ -j\eta(\boldsymbol{r}) & 0 \end{bmatrix} \nabla \times \boldsymbol{E} = -j\eta(\boldsymbol{r}) \nabla E_z \tag{11}$$

$$B = -j\nabla \times \eta(\boldsymbol{r}) \nabla E_z = -j\nabla\eta(\boldsymbol{r}) \times \nabla E_z = (-j\hat{z} \times \nabla\eta(\boldsymbol{r}) . \nabla E_z)\hat{z} \tag{12}$$

$$A = \frac{1}{\tilde{\mu}(\boldsymbol{r})} \nabla \times \nabla \times \boldsymbol{E} + \nabla \frac{1}{\tilde{\mu}(\boldsymbol{r})} \times \nabla \times \boldsymbol{E} \tag{13}$$

$$\nabla \times \nabla \times \boldsymbol{E} = -\nabla^2 \boldsymbol{E} = -\nabla^2 E_z \hat{z} \tag{14}$$

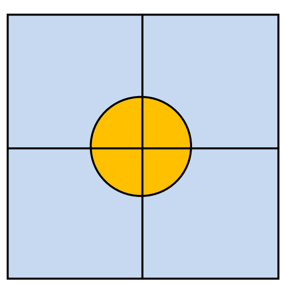
$$\nabla \frac{1}{\tilde{\mu}(\boldsymbol{r})} \times \nabla \times \boldsymbol{E} = \left(\frac{\nabla ln\tilde{\mu}(\boldsymbol{r})}{\tilde{\mu}(\boldsymbol{r})} . \nabla E_z\right)\hat{z} \tag{15}$$

$$A = \left(-\nabla^2 E_z + \frac{\nabla ln\tilde{\mu}(\boldsymbol{r})}{\tilde{\mu}(\boldsymbol{r})} . \nabla E_z\right)\hat{z} \tag{16}$$

$$\frac{1}{\varepsilon(\boldsymbol{r})\tilde{\mu}(\boldsymbol{r})} \left(-\nabla^2 + \left(\nabla \, ln \, \tilde{\mu}\,(\boldsymbol{r}) - \, i \, \tilde{\mu}(\boldsymbol{r}) \, \hat{z} \times \, \nabla\eta(\boldsymbol{r})\right) . \nabla\right) E_z = \frac{\omega^2}{c^2} \, E_z \tag{17}$$

Equation (17) is used for deriving eigen-value equation. For doing this at first we consider a unit cell with one rod. Eigen-value equation with several rods can be derived with a simple shifting of Fourier series.

Figure 1: A unit cell with one rod. The (a) region is inside of the circle and the (b) region is outside of the circle



We assume that $\mu(a) = \mu$, $\mu(b) = 1$, $\tilde{\mu}(a) = \tilde{\mu}$, $\tilde{\mu}(b) = 1$, $\eta(a) = \eta$, $\eta(b) = 0$, $\varepsilon(a) = \varepsilon_a$ and $\varepsilon(b) = \varepsilon_b$. Then we define the following constants:

$$\tilde{\mu}(\boldsymbol{r}) = \alpha\, \eta(\boldsymbol{r}) + const \rightarrow \begin{cases} const = 1 \\ \alpha = \frac{\tilde{\mu}-1}{\eta} \end{cases} \tag{18}$$

$$(\tilde{\mu}\varepsilon)(\boldsymbol{r}) = \beta\eta(\boldsymbol{r}) + const \rightarrow \begin{cases} const = \varepsilon_b \\ \beta = \frac{\tilde{\mu}\varepsilon_a - \varepsilon_b}{\eta} \end{cases} \tag{19}$$

$$ln\, \tilde{\mu}(\boldsymbol{r}) = \gamma\eta(\boldsymbol{r}) + const \rightarrow \begin{cases} const = 0 \\ \gamma = \frac{ln\tilde{\mu}}{\eta} \end{cases} \tag{20}$$

$$\chi = \frac{(\frac{\alpha}{2} \times \eta^{2} + \eta)}{\eta} \tag{21}$$

And the following functions:

$$f\,(\boldsymbol{r}) = ln(\eta(\boldsymbol{r}) + \frac{\varepsilon_b}{\beta}) \tag{22}$$

$$e(\boldsymbol{r}) = \frac{1}{(\varepsilon\tilde{\mu})(\boldsymbol{r})} \tag{23}$$

By considering the above relations we can write the following equations:

$$\frac{1}{(\varepsilon\tilde{\mu})(\boldsymbol{r})}[-\nabla^2 + (\nabla ln\, \tilde{\mu}(\boldsymbol{r}) - j\, \tilde{\mu}(\boldsymbol{r}) \times \nabla\eta(\boldsymbol{r})).\nabla]\, E_z = (\frac{\omega}{c})^2 E_z \tag{24}$$

$$\frac{1}{(\beta\eta(\boldsymbol{r}) + \varepsilon_b)}[-\nabla^2 + (\gamma\nabla\, \eta(\boldsymbol{r}) - j\,(\alpha\, \eta(\boldsymbol{r}) + 1) \times \nabla\eta(\boldsymbol{r})).\nabla]\, E_z = (\frac{\omega}{c})^2 E_z \tag{25}$$

$$\frac{1}{(\beta\eta(\boldsymbol{r}) + \varepsilon_b)}[-\nabla^2 + (\gamma\nabla\, \eta(\boldsymbol{r}) - j \times \nabla(\frac{\alpha}{2} \times \eta^2(\boldsymbol{r}) + \eta(\boldsymbol{r}))).\nabla]\, E_z = (\frac{\omega}{c})^2 E_z \tag{26}$$

$$[-\frac{1}{(\varepsilon\tilde{\mu})(\boldsymbol{r})}\nabla^2 + (\frac{\gamma}{\beta}\frac{\nabla\, \eta(r)}{\left(\eta(r) + \frac{\varepsilon_b}{\beta}\right)} - j \times \frac{\chi}{\beta}\frac{\nabla\, \eta(r)}{\left(\eta(r) + \frac{\varepsilon_b}{\beta}\right)}).\nabla]\, E_z = (\frac{\omega}{c})^2 E_z \tag{27}$$

$$[-\frac{1}{(\varepsilon\tilde{\mu})(\boldsymbol{r})}\nabla^2 + (\frac{\gamma}{\beta}\nabla ln\left(\eta(\boldsymbol{r}) + \frac{\varepsilon_b}{\beta}\right) - j \times \frac{\chi}{\beta}\nabla ln\left(\eta(\boldsymbol{r}) + \frac{\varepsilon_b}{\beta}\right)).\nabla]\, E_z = (\frac{\omega}{c})^2 E_z \tag{28}$$

By using Bloch theorem, we have:

$$E_z(\boldsymbol{r}) = e^{-j\,\boldsymbol{\kappa}.\boldsymbol{r}}\, \Phi(\boldsymbol{r}) \tag{29}$$

In which $\Phi(\boldsymbol{r})$ is a periodic function. Then:

$$[-e(\boldsymbol{r})(\nabla - j\boldsymbol{\kappa})^2 + (\frac{\gamma}{\beta}\nabla f\,(\boldsymbol{r}) - j \times \frac{\chi}{\beta}\nabla f\,(\boldsymbol{r})).(\nabla - j\boldsymbol{\kappa})]\, \Phi(\boldsymbol{r}) = (\frac{\omega}{c})^2 \Phi(r) \tag{30}$$

Because $\varepsilon(\boldsymbol{r})$ and $\mu(\boldsymbol{r})$ are periodic functions, we define 2D Fourier series of $e(\boldsymbol{r}), f(\boldsymbol{r})$ and $\Phi(\boldsymbol{r})$ as follows:

$$e(\boldsymbol{r}) = \sum_{rs} e_{rs}\; e^{-j\,\boldsymbol{G}_{rs}.\boldsymbol{r}} \tag{31}$$

$$f(\boldsymbol{r}) = \sum_{rs} f_{rs} \; e^{-j\,\boldsymbol{G_{rs}}.\boldsymbol{r}} \tag{32}$$

$$\Phi(\boldsymbol{r}) = \sum_{lh} \varphi_{lh} \; e^{-j\,\boldsymbol{G_{lh}}.\boldsymbol{r}} \tag{33}$$

Then $\nabla f(\boldsymbol{r})$ is derived as follows:

$$\nabla f(\boldsymbol{r}) = \sum_{rs} f_{rs} \; e^{-j\,\boldsymbol{G_{rs}}.\boldsymbol{r}}(-j\,\boldsymbol{G_{rs}}) \tag{34}$$

Then we have:

$$\sum_{rslh}\left[e_{rs}(\boldsymbol{G_{lh}}+\boldsymbol{\kappa})^2 - \frac{\gamma}{\beta} f_{rs}\boldsymbol{G_{rs}}.(\boldsymbol{G_{lh}}+\boldsymbol{\kappa}) + \boldsymbol{j}\,\frac{\chi}{\beta} f_{rs}\,\boldsymbol{G_{(-s)r}}.(\boldsymbol{G_{lh}}+\boldsymbol{\kappa})\right]\varphi_{lh}\, e^{-j\,(\boldsymbol{G_{lh}}+\boldsymbol{G_{rs}}).\boldsymbol{r}} = (\frac{\omega}{c})^2\,\Phi(r) \tag{35}$$

We define $(\boldsymbol{G_{lh}}+\boldsymbol{\kappa})^2$, $\boldsymbol{G_{rs}}.(\boldsymbol{G_{lh}}+\boldsymbol{\kappa})$ and $\boldsymbol{G_{(-s)r}}.(\boldsymbol{G_{lh}}+\boldsymbol{\kappa})$ to be A_{rslh}, B_{rslh} and C_{rslh} respectively. Then:

$$A_{rslh} = \left(\frac{2\pi l}{L} + \kappa_x\right)^2 + \left(\frac{2\pi h}{L} + \kappa_y\right)^2 \tag{36}$$

$$B_{rslh} = \frac{2\pi r}{L}\left(\frac{2\pi l}{L} + \kappa_x\right) + \frac{2\pi s}{L}\left(\frac{2\pi h}{L} + \kappa_y\right) \tag{37}$$

$$C_{rslh} = -\frac{2\pi s}{L}\left(\frac{2\pi l}{L} + \kappa_x\right) + \frac{2\pi r}{L}\left(\frac{2\pi h}{L} + \kappa_y\right) \tag{38}$$

$$\sum_{rslh}\left[e_{rs}A_{rslh} - \frac{\gamma}{\beta} f_{rs}B_{rslh} + \boldsymbol{j}\,\frac{\chi}{\beta} f_{rs}\,C_{rslh}\right]\varphi_{lh}\, e^{-j\,(\boldsymbol{G_{lh}}+\boldsymbol{G_{rs}}).\boldsymbol{r}} = (\frac{\omega}{c})^2\,\Phi(r) \tag{39}$$

$$e_{rs}A_{rslh} - \frac{\gamma}{\beta} f_{rs}B_{rslh} + \boldsymbol{j}\,\frac{\chi}{\beta} f_{rs}C_{rslh} = D_{rslh} \tag{40}$$

$$\boldsymbol{G_{lh}} + \boldsymbol{G_{rs}} = \boldsymbol{G_{(l+r),(h+s)}} \tag{41}$$

$$\sum_{rslh} D_{rslh}\,\varphi_{lh}\, e^{-j\,\boldsymbol{G_{(l+r),(h+s)}}.\boldsymbol{r}} = (\frac{\omega}{c})^2 \sum_{lh}\varphi_{lh}\, e^{-j\,\boldsymbol{G_{lh}}.\boldsymbol{r}} \tag{42}$$

We change variable as follows:

$$r + l \to p \qquad s + h \to q \qquad l \to m \qquad h \to n \;\Rightarrow\; r \to p - m \qquad s \to q - n$$

$$S_{mnpq} = D_{p-m,q-n,m,n} \tag{43}$$

$$\sum_{mnpq} S_{mnpq}\,\varphi_{mn}\, e^{-j\,\boldsymbol{G_{mn}}.\boldsymbol{r}} = (\frac{\omega}{c})^2 \sum_{mn}\varphi_{mn}\, e^{-j\,\boldsymbol{G_{mn}}.\boldsymbol{r}} \tag{44}$$

Equation (44) is the eigen-value equation for obtaining eigen-frequencies. By analyzing equation (24) we can see that it is not Hermitian. Thus its eigen-values can be imaginary. Because it is the real part that determines direction of propagation we have sorted eigen-frequencies by their real part to determine which eigen-frequency belong to which band.

3. RESULTS

We have plotted contour-plots for several values of μ and ρ and several setups in which $\varepsilon_a = 15$ and $\varepsilon_b = 1$ were constant, then we see that in some cases contour-plots do not hold any symmetry.

Figure 2: Setup 1, a unit cell with three rods of center position of (0.24,0), (-0.14,0.22) and (-0.14,-0.22) and radius of 0.0635

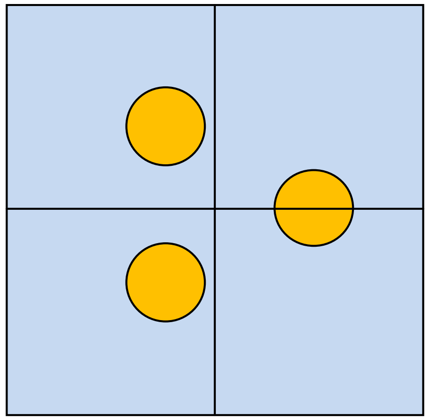


Figure 3: Setup 2, a unit cell with three rods of center position of (0.24,0), (-0.14,0.22) and (0,0) and radius of 0.0635

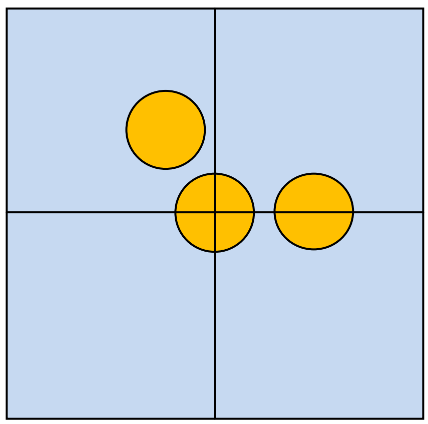


Figure 4: Setup 3, a unit cell with two rods of center position of (0.28,0) and (0.2,0.2) and radius of 0.0778

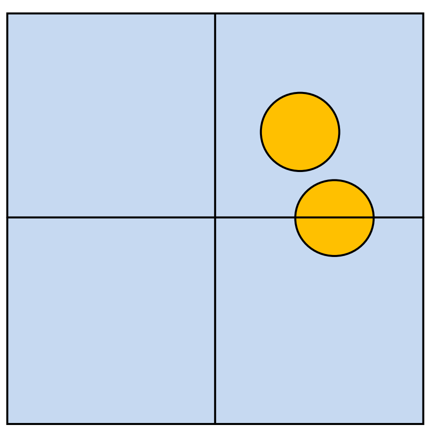


Figure 5: Contour-plot of 2nd band of setup 1 with: $\mu = 30$ and $\rho = 34$

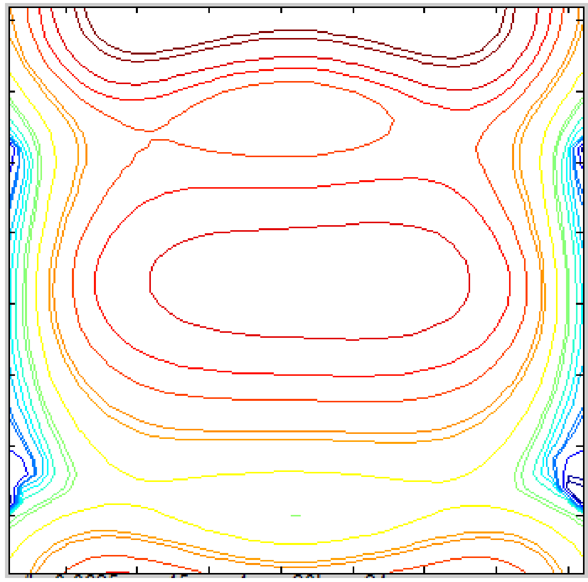


Figure 6: Contour-plot of 3rd band of setup 2 with: $\mu = 40$ and $\rho = 43$

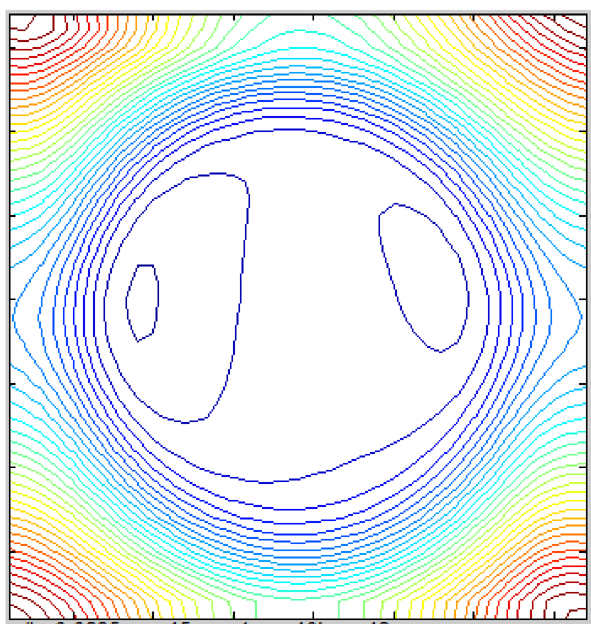


Figure 7: Contour-plot of 1st band of setup 3 with: $\mu = 30$ and $\rho = 36$

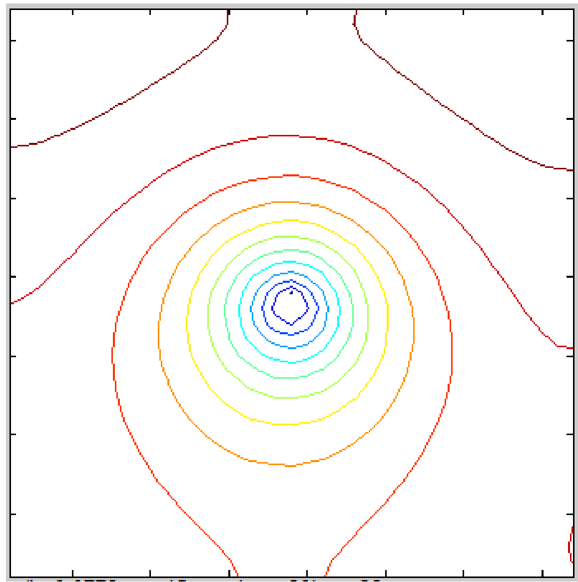


Figure 8: Contour-plot of 2nd band of setup 3 with: $\mu = 30$ and $\rho = 36$

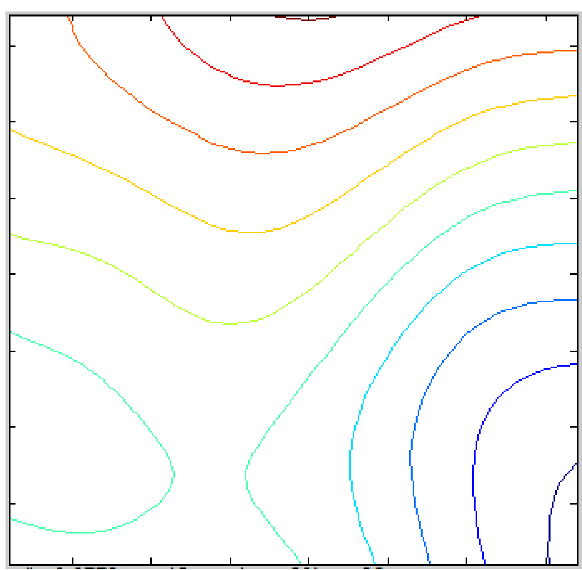


Figure 9: Contour-plot of 4th band of setup 2 with: $\mu = 20$ and $\rho = 22$

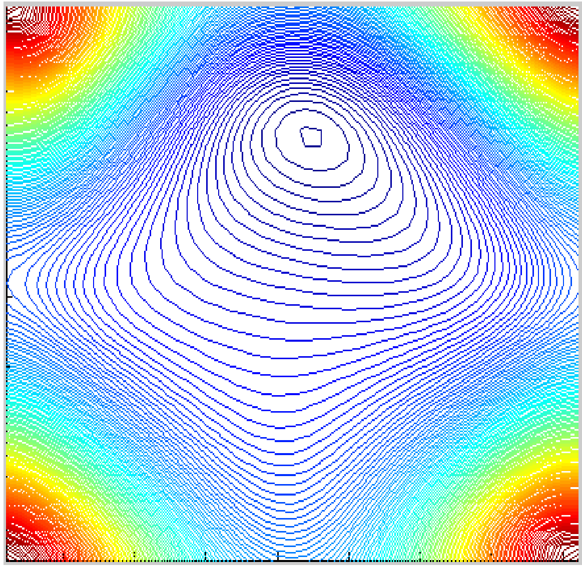
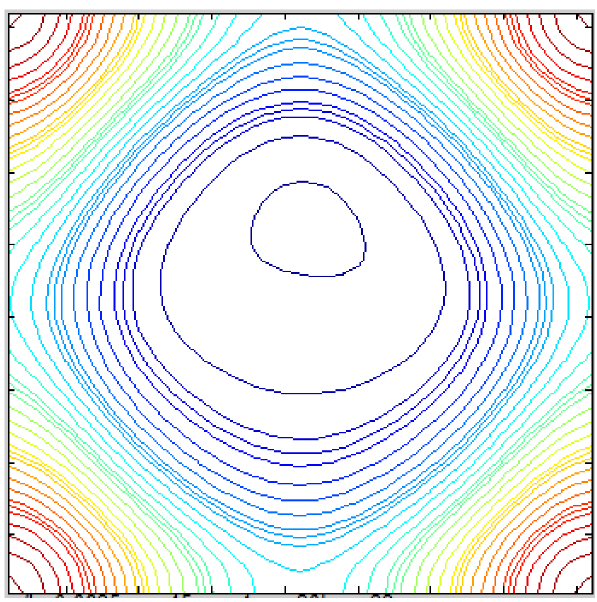


Figure 10: Contour-plot of 4th band of setup 1 with: $\mu = 30$ and $\rho = 33$



4. CONCLUSIONS

For the first time, it is shown that there exist periodic structures in which the irreducible Brillouin Zone completely spans over the whole Brillouin Zone. The symmetry breaking is attained through consideration of both geometrically asymmetry as well as employing constituent materials which violate time-reversal symmetry.

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